Experimental tests and modeling of the optimal orifice size for a closed cycle $^4$He sorption refrigerator

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Abstract

Closed cycle $^4$He sorption refrigerators are an increasingly popular choice for remote cryogenic operations. The presence of a superfluid film on the inner walls of the evaporation pot limits their performance. We present a simple model of the gas, film, and heat flow in a $^4$He sorption refrigerator, taking into account the effects of an orifice to limit superfluid film flow. The model serves as a useful guideline for optimizing cryogenic performance. We also present a diagram of hold time vs. steady state temperature as a function of the radius of the orifice and compare the model to measurements.

Keywords: Sorption coolers (E); Superfluid helium (He II) (B); He II systems (E)

1. Introduction

Closed-cycle sorption refrigeration is advantageous in many fields where the use of expendable cryogens is logistically demanding. For our specific application, the Atacama Cosmology Telescope project [15,12], we use such a system to cool detectors to measure the anisotropy of the cosmic microwave background from Cerro Toco in northern Chile. Our full cryogenic system uses a Cryomech$^1$ pulse tube mechanical refrigerator as the 2.9 K thermal reservoir for the $^4$He sorption refrigerator, which in turn is the 0.6 K reservoir for a pumped $^3$He refrigerator that reaches 0.220 K. Ours and similar systems are described in [9,3,5]. The $^4$He refrigerator, the superfluid film that forms at the lambda transition, $T_\lambda = 2.18$ K, presents technical design challenges that are insufficiently addressed in the literature. In this paper we present the modeling, design, and testing of a pumping orifice that allows one to optimize the ultimate cooling temperature and hold time.

The presence of a superfluid film limits the hold time of many $^4$He refrigerators. The primary influence is to drain liquid $^4$He from the evaporation pot (shown in Fig. 1) and allow it to evaporate in a region of the apparatus where it gives no useful cooling power. The thermal conductivity of the film is negligible [6–8] and does not in itself limit the performance.

To minimize the effects of the superfluid film, a smooth orifice [1,18] is introduced into the pump tube, just above the evaporation pot. This limits the amount of helium that will be lost by bulk transport since the flow velocity of the film is bounded above by a superfluid critical velocity and the finite film thickness which is determined by the surface properties [23]. This orifice can dramatically increase the hold time, $t_{\text{hold}}$, of the refrigerator. However, if the radius $a$ of the constriction is too small, the steady state temperature, $T_\text{0}$, of the evaporation pot will suffer because the pumping speed is reduced. We present below a model that describes the dependance of $t_{\text{hold}}$ and $T_\text{0}$ on $a$. To describe

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the system with just a simple set of equations, we make a number of approximations which are outlined. Rather than describing the details of the gas and fluid flow, the goal of the model is to provide guidelines for building and optimizing related systems. In particular, we determine the value of $a$ that gives the best performance, that is, the longest hold time $t_{\text{hold}}$ without the pot temperature $T_0$ exceeding the desired design temperature.

2. Design of the $^4$He sorption refrigerator

Fig. 1 shows the sorption refrigerator used for testing. The lower part of apparatus, the evaporator pot, contains liquid helium at a steady state temperature of $T_0$ and pressure $P_0 = P_{\text{vap}}(T_0)$. An orifice with a small hole of radius $a = 6.35 \times 10^{-3}$ m, length $L_p = 0.130$ m, and is
manufactured from 316 stainless steel with a wall thickness of $t_p = 2.54 \times 10^{-4}$ m. At the top of the pump tube is the activated charcoal pump. At a distance $L_c = 0.0866$ m above the constriction, a condensation plate (CP) is soldered to the pump tube. The CP is held at temperature, $T_c$, by the pulse tube refrigerator at the end of the condensation plate. Though $T_c$ varies somewhat throughout the cycle, it settles to a constant value at the end of the condensation phase permitting the determination of the amount of condensed $^4$He. The superfluid helium film creeps up to a distance $l$ below the condensation plate, where we assume evaporation takes place at a fixed height. We label the pressure just above the diaphragm by $P_1$ and the pressure at the top of the film by $P_2$. The whole system is closed, containing $n_0 = 1.64$ mol of 99.999% $^4$He corresponding to a pressure of $5.17 \times 10^6$ N/m$^2$ at 298 K. Three such refrigerators were made with different orifice radii $a = 508 \mu$m, 572 $\mu$m, 635 $\mu$m.

3. Model of refrigerator

The model of the gas and film flow through the apparatus breaks down into three parts: the flow of gas through the constriction, of gas along the pump tube, and of the film through the constriction (and up the wall of the pump tube). By necessity the treatment of the rarefied gas is approximate, since the gas is highly compressible fluid and in the transition regime between continuous behavior and free molecular flow. In addition, the gas-superfluid interface is incompletely understood at a fundamental level. However, though the physical processes involved in the refrigerator are in general complex, we find that with a number of simplifying assumptions, we can model the gross characteristics of the system.

The mass transport from the pot through the constriction is divided between the gas and film. Let us denote the corresponding rates by $dM_1/dt$ and $dM_2/dt$. The film then creeps up the inner wall of the pump tube, and evaporates before it reaches the condensation plate, since the latter is kept at a constant temperature $T_c > T_s$. We shall assume that the evaporation takes place at a distance $l$ from the condensation plate (rather than spread out over a certain area) and that the film evaporates at a temperature $T_s$.

Given these assumptions, the gas and film flow can be described by four mass transport rates: the flow of the gas through the orifice, $dM_1/dt$, the flow of film through the orifice, $dM_2/dt$, the flow of gas through the lower part of the pump tube, $dM_3/dt$, and the gas flow in the upper part of the pump tube (above the point where the film evaporates) $dM_4/dt$. These flow rates must satisfy

$$dM_3/dt = dM_1/dt,$$

$$dM_4/dt = dM_1/dt + dM_2/dt.$$  

3.1. Relevant properties of $^4$He gas

At temperatures above its boiling point the stable phase of helium is a monatomic and very weakly interacting gas, that over a wide range of temperatures can be described as an ideal gas with a ratio of heat capacities of $\gamma = 5/3$.

The viscosity of an ideal gas is given by the kinetic theory expression, $\eta = \frac{5}{16\pi} \sqrt{\frac{k_B T m}{\pi}}$ where $d$ is the effective diameter of a gas molecule, $k_B$ is Boltzmann’s constant, $T$ is the temperature of the gas, and $m$ is the atomic mass of the gas, which is based on the assumption of elastic collisions between gas molecules of effective diameter $d$ without long range interactions. This formula does not apply exactly to real gases, and at temperatures close to its boiling point even the weak Van der Waals interactions of $^4$He must be included. Various empirical formulas with modified temperature dependence [10] have been proposed. In the absence of comprehensive data on the viscosity of $^4$He gas below its condensation temperature, we shall nevertheless use the ideal gas formula, and choose $d$ such that the viscosity at $T = 1.64$ K agrees with the value of $\eta = 5.47 \times 10^{-5}$ kg m$^{-1}$ s$^{-1}$ measured by van Itterbeeck and Keesom [13]. This implies $d = 3.54 \times 10^{-10}$ m.

The mean free path $\lambda$ is related to the viscosity through an equation of the form $\eta \propto \rho g \sqrt{\lambda}$, where the density of the gas $\rho g$ is given by $\rho g = \frac{m}{c_s^2}$ and the velocity of sound by $c_s = \sqrt{\frac{2 k_B T}{m}}$. Purely geometrical considerations allow us to express $\lambda$ in terms of the effective diameter $d$: $\lambda = \frac{2 m}{c_s^2 \rho g d^2}$.

For a realistic base temperature of $T_0 = 0.6$ K and assuming saturated vapor pressure this implies $\lambda = 4 \times 10^{-4}$ m, which is of the same order of magnitude as the radius $a$ of the constriction. Thus we are right in the transition regime between continuous fluid flow and free molecular flow.

3.2. Gas flow through the orifice

Since we shall not be dealing with temperatures much below $T_0 = 0.6$ K or constrictions of radius much less than $a = 10^{-4}$ m we shall use fluid dynamics to describe the flow through the orifice. We will show that assuming free molecular flow leads to a similar expression for the flow rate.

To determine the gas flow through the constriction, $dM_1/dt$, we use the fact that energy is conserved for an ideal gas in steady adiabatic flow [11],

$$\frac{\gamma}{\gamma - 1} \frac{P}{\rho g} + gz + \frac{1}{2} u^2 = c_p T + gz + \frac{1}{2} u^2 = \text{constant},$$  

where $g$ is the gravitational acceleration, $z$ is the height, and $u$ is the flow velocity of the gas. Given the temperature $T_0$ and the vapor pressure $P_0 = P_{\text{vap}}(T_0)$ of the gas in the evaporation pot away from the constriction where $u \approx 0$, the constant in Eq. (3) is found to be $(\gamma/(\gamma - 1))P_0/\rho_g(T_0, P_0)$. 

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The gravitational potential term is negligible, and using the adiabaticity constraint \( \rho \propto P^{1/\gamma} \) [11, p. 82] we find that \( \gamma \) at the orifice reaches the local velocity of sound, when the pressure at the orifice takes the critical value \( P_c = P_0(2/\gamma + 1)_{1/2}(\gamma - 1) \). At this point the mass flow rate takes its maximum value, and reducing the pressure \( P_1 \) just downstream of the constriction below the critical pressure will not increase it any further.

If \( P_1 \leq P_c \) the flow is said to be choked. The physics of choked flow involves finite pressure steps across shock fronts and is relevant to the design of nozzles and rockets. Defining \( P_1' = \text{Max}(P_1, P_c) \) we can write the resulting mass flow rate [11] as

\[
\frac{dM_1}{dr} = \pi a^2 \rho(T_o, P_0) c(T_o) \left( \frac{2}{\gamma - 1} \right) \left( \frac{P_1}{P_0} \right)^{\frac{\gamma - 1}{2}} 
\]

(4)

If we had done the same calculation of the mass flow rate through a small orifice assuming free molecular flow and furthermore negligibly small \( P_1 \), we would have obtained the expression

\[
\frac{dM_1}{dr} = \pi a^2 \rho(T_o, P_0) c(T_o) \sqrt{\frac{1}{2\pi\gamma}} 
\]

(5)

Apart from the numerical coefficient, this expression is identical to Eq. (4) in the regime of choked flow, but its validity is restricted to cases where \( \lambda \gg a \) and thus we shall employ Eq. (4) in what follows.

3.3. Gas flow through the pump tube

To find an expression for the mass flow rate of gas through the pump tube above the orifice, we postulate that the gas that evaporates in the evaporator pot flows up the tube at constant temperature \( T_0 \) until it reaches the top of the helium film. Beyond that point all the gas, including the evaporation from the film, flows at a constant temperature \( T_s \) towards the pump. While these assumptions are approximate, the details of the temperature distribution of the gas are of secondary importance.

The flow rate in a generic tube of radius \( b \), length \( l' \), and pressures \( P_1 \) and \( P_2 \) at the high and low pressure ends can be approximated [10] as

\[
\frac{dN}{dr} = \frac{1}{\pi b} \left( \frac{P_1 - P_2}{2} \right) \left( \frac{P_1 + P_2}{2} \right) \left( \frac{\pi b^4}{8 \eta l'} \right) \left( \frac{P_1 - P_2}{P_0^2} \right),
\]

(6)

where \( \eta \) is the viscosity of the gas and \( m \) is the atomic mass. This expression is approximately valid both in the regime of continuous, viscous flow and in the regime of free molecular flow, and interpolates correctly between them. Thus, if \( P_1 \) is the pressure just downstream of the constriction, and \( P_2 \) the pressure at the top of the film, a distance \( l \) below the condensation plate, and a distance \( L_c - l \) above the orifice, the transport rate of the gas that evaporated in the pot is given by

\[
\frac{dM_3}{dr} = \frac{m}{k_B T_0} \left( \frac{\pi b^4}{8 \eta (T_0)(L_c - l)} \left( \frac{P_1 - P_2}{2} \right) + 2\pi \frac{b^3}{3} \left( \frac{8 \pi m b^3}{P_0} \right)^{\frac{1}{2}} (P_1 - P_2) \right).
\]

(7)

Similarly, taking \( P = 0 \) at the upper end of the pump tube, which is a distance \( L_p \) above the constriction, the mass flow rate of gas between the upper end of the film and the pump is

\[
\frac{dM_4}{dr} = \frac{m}{k_B T_s} \left( \frac{\pi b^4}{8 \eta (T_s)(L_p - L_c + l)} \left( \frac{P_1 - P_2}{2} \right) + 2\pi \frac{b^3}{3} \left( \frac{8 \pi m b^3}{P_0} \right)^{\frac{1}{2}} (P_1 - P_2) \right).
\]

(8)

3.4. Film flow through the constriction

The two-fluid model of superfluid \( ^4 \)He asserts that He II is composed of a superfluid component of density, \( \rho_s \), and a normal fluid of density, \( \rho_n \). Any surface in contact with \( ^4 \)He vapor at \( T < T_s \) will be covered by a film of liquid helium [19,4,2]. The equilibrium thickness of the film is essentially determined by the Van der Waals forces exerted by the molecules of the wall. The condition that the free surface of the film has to be at the same chemical potential leads to a Bernoulli type equation for the film not unlike Eq. (3):

\[
\frac{1}{2} \frac{\rho_s}{\rho_0} u_s^2 + \frac{P}{\rho_0} - \sigma T + gz + \frac{x}{\rho_0} = \text{constant},
\]

(9)

where \( u_s \) is the superfluid flow velocity, \( \sigma \) the specific entropy, \( t \) the thickness of the film, \( \rho_s \) the superfluid component of the density, \( \rho_0 = \rho_s + \rho_n \approx 145 \text{ kg m}^{-3} \) [22] the density of liquid helium at superfluid temperatures, \( n_1 = 3 \) the Van der Waals exponent [20,14,21], and \( x \) a constant quantifying the Van der Waals interaction.

The thickness \( t \) of an isothermal and isobaric film measured a height \( z \) from the free surface of the bulk liquid is given by \( t(z) = \left( \frac{z}{z_0} \right)^{1/3} \). We assume that at the upper walls of the evaporator pot, where the film is almost static, and begins to converge radially to the constriction, the film has a thickness \( t_0 \), which in turn fixes the value of \( x \). We take \( t_0 = 30 \text{ nm} \), which for \( T < T_s \) is a typical value for pure \(^4 \)He on a clean, smooth substrate [20,23].

Consider now the moving film [14,21]. The film attached to the lower surface of the diaphragm moves radially towards the constriction, at a constant temperature and gravitational potential. Far from the orifice, the thickness is \( t_0 \), and the pressure \( P_0 \), so applying Eq. (9)

\[
t = t_0 \left( 1 + \frac{t_0}{x \rho_0} \left( \frac{1}{2} \rho_s u_s^2 + P - P_0 \right) \right)^{\frac{1}{4}}.
\]

(10)
The mass flow rate of the film \( \frac{dM_f}{dt} = 2\pi r(t)u_c(r)\rho_l \) must be independent of \( r \). The final ingredient required to calculate its value is the flow velocity. It has been shown there is an upper limit to the velocity of superfluid film flow characterized by the sudden onset of dissipation that destroys superfluidity [20]. We use this maximum velocity \( u_c \) as the one fit parameter of our model for comparing to experimental data. Values generally lie in the range 0.1 < \( u_c < 1 \, \text{m s}^{-1} \) [20,23].

Assuming that evaporation at the top of the film always proceeds sufficiently rapidly to drive the film through the orifice at the maximum rate \( u_c \),

\[
\frac{dM_f}{dt} = 2\pi a_0 u_c(T_0) \left( 1 + \frac{t_a^3}{2\rho_l} \frac{1}{2}\rho_l u_c^2 + P_1 - P_0 \right)^{-1/2}.
\]

(11)

With expressions for the four relevant mass flow rates \( dM_f/dt \), and given values of \( T_0 \) and \( l \) we can impose Eqs. (1) and (2) to calculate the total mass loss rate, and thus the hold time. We next consider heat transport along the pump tube to find \( T_0 \).

3.5. Heat flow along pump tube

The walls of the stainless steel pump tube determine the load on the evaporation pot. Knowing the thermal conductivity \( \kappa \) of stainless steel as a function of temperature, the total heat conducted through a distance \( l \) from the condensation plate at temperature \( T_c \) to the top of the film is

\[
W_{tot} = \frac{2\pi h t_p}{l} \int_{T_c}^{T_s} \kappa(T) \, dT,
\]

(12)

where \( t_p \) is the wall thickness of the pump tube, \( \kappa(T) \) is the thermal conductivity of stainless steel [16] and the top of the film is assumed to be at \( T_s \).

The remainder of the heat flow, that is the part that is not used to evaporate the film, is conducted through a distance \( L_c - l \) along the lower part of the pump tube and into the liquid helium bath. The power into the evaporation pot, which is at temperature \( T_0 \), is therefore given by

\[
W_{pot} = \frac{2\pi h t_p}{L_c - l} \int_{T_s}^{T_c} \kappa(T) \, dT.
\]

(13)

This is the power that evaporates \( ^4\text{He} \) from the free surface of the bulk liquid.

The power into the evaporator pot serves to evaporate helium at a rate \( dM_1/dt \) and at a temperature \( T_0 \). Thus,

\[
W_{pot} = \frac{dM_1}{dt} \frac{L(T_0)}{mN_A},
\]

(14)

where \( L \) is the latent heat of helium [17], and \( N_A \) is Avagadro’s number. The total power drawn from the condensation plate includes \( W_{pot} \) as well as the power required to provide the latent heat to evaporate the film at temperature \( T_c \). Therefore,

\[
W_{tot} = \frac{dM_1}{dt} \frac{L(T_0)}{mN_A} + \frac{dM_2}{dt} \frac{L(T_s)}{mN_A}.
\]

(15)

Combining the constraints (1), (2), (14) and (15) with the relations (4), (7), (8) and (11)–(13), the hold times and steady state temperatures as functions of the specifications of the refrigerator can be computed, in particular as functions of \( a \). The variables \( T_0, P_1, P_2 \) and \( l \) are found through the consistency of the equations.

3.6. Applying the model

The model has been applied to the refrigerator depicted in Fig. 1.

Not all of the \(^4\text{He} \) in the refrigerator can condense. Some of the helium remains gaseous and some fraction of the condensed helium is subsequently spent on the initial cool down. The amount of helium usable for steady state refrigeration is a strong function of \( T_c \). Ideally, as much as 0.8\% is available.

Agreement between the model and data is found when \( u_c = 1.2 \, \text{m s}^{-1} \). While slightly larger than expected, we believe that in reality the film is thicker than \( t_0 = 30 \, \text{nm} \) due to the approximately 0.05 \( \mu \text{m Ra} \) surface roughness of the polished stainless orifice, and that the true \( u_c \) value is lower. It has been shown that when He II flows over a rough surface its thickness is larger than when flowing over a smooth surface [20]. We also find for no external load on the refrigerator, \( a = 635 \, \mu\text{m}, T_0 = 0.612 \, \text{K}, P_1 = 0.047 \, \text{N/m}^2, P_2 = 0.046 \, \text{N/m}^2 \) and \( l = 2 \, \text{mm} \), in agreement with expectations. With a condensation plate temperature of \( T_c = 4.2 \, \text{K} \), the hold time is 40 h. With \( T_0 = 2.9 \, \text{K} \) the hold time is 90 h at \( T_0 = 0.612 \, \text{K} \). Since the pressure drop across the stainless steel pump tube is small and the thermal conductivity of stainless is small, the pump tube may likely be made much shorter but we have not investigated these geometries.

Fig. 2 plots data for the three values of \( a \) and the subsequent model fit.

From Fig. 2 it is evident that the optimal orifice size is determined by \( T_0 \), loading, and the desired hold time. As expected, for fixed pot temperature the hold time is a monotonically decreasing function of the constriction radius \( a \), since the matter transport rates through the constriction for the film increase with \( a \). The steady state temperature however, somewhat surprisingly exhibits a shallow minimum.

For small \( a \) (\( a < 570 \, \mu\text{m} \) for no external loading) as \( a \) decreases \( T_0 \) increases since not enough helium can be removed from the evaporator pot to cool it efficiently. The behavior for large orifice radii, on the other hand, is more complicated and leads to the “hooks” on the lines

\(^2 \text{Ra} \) is an industry standard specification for average surface roughness. For most machining processes, the rms roughness is 1.3 times the Ra value. We give Ra so one can make a direct comparison to standard roughness gauges.
of constant power loading. In increasing $a$ starting from the minimum at $a = 570$ µm (for our setup with no external load), the total mass loss rate increases. Thus a larger pressure drop across the pump tube is required to transport the gas to the pump, and $P_1$ increases. However, $P_1$ can never rise above the saturated vapor pressure $P_0$ of the helium in the evaporator pot, or else no gas could be transported through the orifice into the pump tube. Combined with the exponential temperature variation of the vapor pressure of helium, this sets a lower limit on $T_0$ which increases with $a$. However, this limitation could easily be overcome by using a wider pump tube to reduce the pressure drop.

We routinely operate four systems based on the design with $a = 508$ µm. With $T_c = 2.9$ K, the evaporation pot remains at 0.61 K for 98 h with no external loading. The same target is reached on each cycle to ±10 mK. The system is not noticeably sensitive to vibration suggesting that the copper foam effectively connects the $^4$He liquid to the evaporation pot.